On Completely ρ -Irresolute and Weakly ρ -Irresolute Functions

DEVAMANOHARAN CHITHARANJAN* AND PIOUS MISSIER

Post Graduate and Research Department of Mathematics, V. O. Chidambaram College, Thoothukudi-628 008, Tamil Nadu, India

e-mail: kanchidev@gmail.com; spmissier@yahoo.com

ABSTRACT. The purpose of this paper is to introduce two new types of irresolute functions called, completely ρ -irresolute functions and weakly ρ -irresolute functions. We obtain their characterizations and their basic properties.

1. Introduction and preliminaries

Functions and of course irresolute functions give new path towards research. In 1972, Crossley and Hildebrand [2] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years, for example see [7,8,10]. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences.

Recently, as generalization of closed sets, the notion of ρ -closed sets were introduced and this notion was further studied by the authors [3,4,5]. In this paper, We will continue the study of related irresolute functions with ρ -open sets. We introduce and characterize the concepts of completely ρ -irresolute functions and weakly ρ -irresolute functions.

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned and $f: (X, \tau) \rightarrow (Y, \sigma)$ (or simply $f: X \rightarrow Y$) denotes a function f of a space (X, τ) into space (Y, σ) . Let A be a subset of a space X . The closure and the interior of A are denoted by $\text{Cl}(A)$ and $\text{Int}(A)$, respectively.

Definition 1.1. Let (X, τ) be a topological space. A subset A of the space X is said to be

1. preopen [17] if $A \subseteq \text{int}(\text{cl}(A))$ and preclosed [17] if $\text{cl}(\text{int}(A)) \subseteq A$.
2. semi-open [14] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed [1] if $\text{int}(\text{cl}(A)) \subseteq A$.
3. regular open [21] if $A = \text{int}(\text{cl}(A))$ and regular closed [21] if $A = \text{cl}(\text{int}(A))$.

* Corresponding Author.

Received March 12, 2011; accepted September 23, 2011.

2010 Mathematics Subject Classification: 54C10, 54C08, 54C05.

Key words and phrases: Topological spaces, ρ -open sets, ρ -irresolute functions.

Definition 1.2([7]). Let (X, τ) be a topological space and $A \subseteq X$.

1. The pre-interior of A , denoted by $\text{pint}(A)$, is the union of all preopen subsets of A .
2. The pre-closure of A , denoted by $\text{pcl}(A)$, is the intersection of all preclosed sets containing A .

Definition 1.3. Let (X, τ) be a topological space. A subset $A \subseteq X$ is said to be

1. $\hat{g}$ -closed [23] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X .
2. *g -closed [25] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ -open in X .
3. $\#g$ - semi closed (briefly $\#gs$ -closed)[24] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is *g -open in X .
4. $\tilde{g}$ -closed set [12] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#gs$ -open in X .
5. ρ -closed set [3] if $\text{pcl}(A) \subseteq \text{Int}(U)$ whenever $A \subseteq U$ and U is $\tilde{g}$ -open in X .

The complements of the above mentioned sets are called their respective open sets.

The family of all ρ -open subsets of (X, τ) is denoted by $\rho O(X)$. We set $\rho O(X, x) = \{V \in \rho O(X) | x \in V\}$ for $x \in X$.

The intersection of all ρ -closed sets containing a set A in a space X is called the ρ -closure of A and is denoted by $\rho\text{-cl}(A)$ [3].

Theorem 1.4([3]). *Every preclosed subset of X is ρ -closed.*

Lemma 1.5([3]). *If A is a ρ -closed set then $\rho\text{-cl}(A) = A$. Converse need not be true.*

Theorem 1.6([3]). *A subset A of (X, τ) is regular open if A is both open and ρ -closed.*

Definition 1.7. A space (X, τ) is called locally indiscrete space [18] if every open subset of X is closed.

Definition 1.8. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called:

1. strongly continuous [13] if $f^{-1}(V)$ is both open and closed in X for each subset V of Y .
2. perfectly ρ -continuous [4] if $f^{-1}(V)$ is clopen in X for every ρ -closed (resp. ρ -open) subset V of Y .
3. ρ -irresolute [3] if $f^{-1}(V)$ is ρ -closed (resp. ρ -open) in X for every ρ -closed (resp. ρ -open) subset V of Y .
4. contra- ρ -irresolute if $f^{-1}(V)$ is ρ -open (resp. ρ -closed) in X for every ρ -closed (resp. ρ -open) subset V of Y .

2. Completely ρ -irresolute functions

Definition 2.1. A function $f : X \rightarrow Y$ is called completely ρ -irresolute if the inverse image of each ρ -open subset of Y is regular open in X .

Example 2.2. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and

$\sigma = \{\phi, \{a\}, \{c\}, \{c, a\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a$; $f(b) = f(c) = c$. Then the function f is completely ρ -irresolute.

Theorem 2.3. *Every strongly continuous function is completely ρ -irresolute.*

Proof. Let V be ρ -open subset of Y . By hypothesis, $f^{-1}(V)$ is both open and closed in X . Since $f^{-1}(V)$ is preclosed and by Theorem 1.4, $f^{-1}(V)$ is ρ -closed in X and also by Theorem 1.6, $f^{-1}(V)$ is regular open in X . Hence f is completely ρ -irresolute. $\square$

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.4. Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$ with topologies $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{c\}, \{a\}, \{c, a\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = f(b) = f(d) = b$; $f(c) = c$ is completely ρ -irresolute but not Strongly continuous.

Theorem 2.5. *Every completely ρ -irresolute function on a locally indiscrete space is contra- ρ -irresolute.*

Proof. Let V be ρ -open subset of Y . By hypothesis, $f^{-1}(V)$ is regular open in X . Since X is a locally indiscrete space, $f^{-1}(V)$ is closed and hence it is preclosed in X . Then by Theorem 1.4, $f^{-1}(V)$ is ρ -closed in X and so f is contra- ρ -irresolute. $\square$

The converse of the above Theorem need not be true in general as seen from the following example.

Example 2.6. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra- ρ -irresolute but not completely ρ -irresolute.

Theorem 2.7. *The following statements are equivalent for a function $f : X \rightarrow Y$*

- (i) *f is completely ρ -irresolute;*
- (ii) *$f^{-1}(F)$ is regular closed in X for every ρ -closed set F of Y .*

Proof. (i) $\Rightarrow$ (ii): Let F be any ρ -closed set of Y . Then $Y \setminus F$ is ρ -open set in Y . By (i), $f^{-1}(Y \setminus F) = X - f^{-1}(F)$ is regular open in X . We have $f^{-1}(F)$ is regular closed in X .

Converse is similar. $\square$

Lemma 2.8([5]). *Let S be an open subset of a space (X, τ) . Then the following hold:*

- (i) *If U is regular open in X , then so is $U \cap S$ in the subspace (S, τ_s) .*
- (ii) *If $B \subset S$ is regular open in (S, τ_s) . then there exists a regular open set U in (X, τ) such that $B = U \cap S$.*

Theorem 2.9. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a completely ρ -irresolute function and A is any open subset of X , then the restriction $f|_A : A \rightarrow Y$ is completely ρ -irresolute.*

Proof. Let F be a ρ -open subset of Y . By hypothesis, $f^{-1}(F)$ is regular open in X .

Since A is open in X , it follows from the previous lemma that $(f|_A)^{-1}(F) = A \cap f^{-1}(F)$, which is regular open in A . Therefore, $f|_A$ is completely ρ -irresolute. $\square$

Theorem 2.10. *The following hold for function $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$:*

- (i) *If f is completely ρ -irresolute and g is perfectly ρ -continuous, then $g \circ f$ is completely ρ -irresolute.*
- (ii) *If f is completely ρ -irresolute and g is ρ -irresolute, then $g \circ f$ is completely ρ -irresolute.*
- (iii) *If f is completely ρ -irresolute and g is strongly continuous functions, then $g \circ f$ is completely ρ -irresolute.*

Proof. The proof of the theorem is obvious and hence omitted. $\square$

Definition 2.11. A space X is said to be ρ -connected [4] (resp. almost connected [9]) if there does not exist disjoint ρ -open (resp. regular open) sets A and B such that $A \cup B = X$.

Theorem 2.12. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely ρ -irresolute surjective function and X is almost connected, then Y is ρ -connected.*

Proof. Suppose that Y is not ρ -connected, Then there exist disjoint ρ -open sets A and B of Y such that $A \cup B = Y$. Since f is completely ρ -irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X . Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \emptyset$ and $f^{-1}(B) \neq \emptyset$. This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected. By contradiction, Y is ρ -connected. $\square$

Definition 2.13. A space X is said to be

- (i) nearly compact [19, 20] if every regular open cover of X has a finite subcover;
- (ii) nearly Lindelof [9] if every cover of X by regular open sets has a countable subcover;
- (iii) nearly countably compact [11] if every countable cover by regular open sets has a finite subcover;
- (iv) ρ -compact [4] if every ρ -open cover of X has a finite subcover;
- (v) countably ρ -compact if every ρ -open countable cover of X has a finite subcover;
- (vi) ρ -Lindelof if every cover of X by ρ -open sets has a countable subcover.

Theorem 2.14. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a completely ρ -irresolute surjective function. Then the following statement hold:*

- (i) *If X is nearly compact, then Y is ρ -compact.*
- (ii) *If X is nearly Lindelof, then Y is ρ -Lindelof.*
- (iii) *If X is nearly countably compact, then Y is countably ρ -compact.*

Proof. (i) Let $f : X \rightarrow Y$ be a completely ρ -irresolute function of nearly compact space X onto a space Y . Let $\{U_\alpha : \alpha \in \Delta\}$ be any ρ -open cover of Y . Then, $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X . Since X is nearly compact, there exists a finite

subfamily, $\{f^{-1}(U_{\alpha_i}) | i=1,2,\dots,n\}$ of $\{f^{-1}(U_{\alpha}) : \alpha \in \Delta\}$ which cover X . It follows that $\{U_{\alpha_i} : i = 1,2,\dots,n\}$ is a finite subfamily of $\{U_{\alpha} : \alpha \in \Delta\}$ which cover Y . Hence, space Y is a ρ -compact space.

The proof of other cases are similar. $\square$

Definition 2.15. A space (X, τ) is said to be:

- (i) ρ -closed compact (resp. S -closed [22]) if every ρ -closed (resp. regular closed) cover of X has a finite subcover;
- (ii) countably ρ -closed compact (resp. countably S -closed compact [6]) if every countable cover of X by ρ -closed (resp. regular closed) sets has a finite subcover;
- (iii) ρ -closed Lindelof (resp. S -Lindelof [16]) if every cover of X by ρ -closed (resp. regular closed) sets has a countable subcover.

Theorem 2.16. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a completely ρ -irresolute surjective function. Then the following statements hold:

- (i) If X is S -closed, then Y is ρ -closed compact.
- (ii) If X is S -Lindelof, then Y is ρ -closed Lindelof.
- (iii) If X is countably S -closed-compact, then Y is countably ρ -closed compact.

Proof. (i) Let $f : X \rightarrow Y$ be a completely ρ -irresolute surjective function. Let $\{U_{\alpha} : \alpha \in \Delta\}$ be any ρ -closed cover of Y . Then, by Theorem 2.7, $\{f^{-1}(U_{\alpha}) : \alpha \in \Delta\}$ is a regular closed cover of X . Since X is S -closed, there exists a finite subfamily, $\{f^{-1}(U_{\alpha_i}) | i=1,2,\dots,n\}$ of $\{f^{-1}(U_{\alpha}) : \alpha \in \Delta\}$ which cover X . It follows that $\{U_{\alpha_i} : i = 1,2,\dots,n\}$ is a finite subfamily of $\{U_{\alpha} : \alpha \in \Delta\}$ which cover Y . Hence, space Y is a ρ -closed compact space.

The proof of other cases are similar. $\square$

Definition 2.17. A space (X, τ) is said to be ρ - T_1 (resp. r - T_1 [9]) if for each pair of distinct points x and y of X , there exist ρ -open (resp. regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 2.18. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely ρ -irresolute injective function and Y is ρ - T_1 , then X is r - T_1 .

Proof. Let x, y be any distinct points of X . Since f is injective, we have $f(x) \neq f(y)$. Since Y is ρ - T_1 , there exist ρ -open sets V and W in Y such that $f(x) \in V$, $f(y) \in W$, $f(x) \notin W$ and $f(y) \notin V$. Since f is completely ρ -irresolute, $f^{-1}(V)$ and $f^{-1}(W)$ are regular open sets in X such that $x \in f^{-1}(V)$, $y \in f^{-1}(W)$, $x \notin f^{-1}(W)$ and $y \notin f^{-1}(V)$. This shows that X is r - T_1 . $\square$

Definition 2.19. A space (X, τ) is said to be ρ - T_2 if for each pair of distinct points x and y in X , there exist disjoint ρ -open sets U_1 and U_2 in X such that $x \in U_1$ and $y \in U_2$.

Theorem 2.20. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is completely ρ -irresolute injective function and Y is ρ - T_2 , then X is T_2 .

Proof. Let x, y be any distinct points of X . Since f is injective, we have $f(x) \neq f(y)$.

Since Y is ρ - T_2 , there exist ρ -open sets V and W in Y such that $f(x) \in V$, $f(y) \in W$ and $V \cap W = \phi$. Since f is completely ρ -irresolute, $f^{-1}(V)$ and $f^{-1}(W)$ are regular open sets in X such that $x \in f^{-1}(V)$, $y \in f^{-1}(W)$ and $f^{-1}(V) \cap f^{-1}(W) = \phi$. This shows that X is T_2 . $\square$

Definition 2.21. A function $f : X \rightarrow Y$ is called ρ^* -closed [5] if the image of each ρ -closed set of X is a ρ -closed set in Y .

Theorem 2.22. If a mapping $f : X \rightarrow Y$ is ρ^* -closed, then for each subset B of Y and each ρ -open set U of X containing $f^{-1}(B)$, there exists a ρ -open set V in Y containing B such that $f^{-1}(V) \subset U$.

Proof. Suppose that f is ρ^* -closed map. Let $B \subset Y$ and U be ρ -open set of X such that $f^{-1}(B) \subset U$. Since f is ρ^* -closed, $(f(U^c))^c = V$ is a ρ -open set in Y containing B such that $f^{-1}(V) \subset U$. $\square$

Definition 2.23. A space X is said to be strongly ρ -normal (resp. mildly ρ -normal) if for each pair of distinct ρ -closed (resp. regular closed) sets A and B of X , there exist disjoint ρ -open sets U and V such that $A \subset U$ and $B \subset V$.

Theorem 2.24. If $f : X \rightarrow Y$ is completely ρ -irresolute, ρ^* -closed function from a mildly ρ -normal space X onto a space Y , then Y is strongly ρ -normal.

Proof. Let A and B be two disjoint ρ -closed subsets of Y . Since f is completely ρ -irresolute, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X . Since X is mildly ρ -normal space, there exist disjoint ρ -open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is ρ^* -closed, $f(X \setminus U)$ and $f(X \setminus V)$ are ρ -closed sets in Y . Then by Theorem 2.22, there exist ρ -open sets $G = Y \setminus f(X \setminus U)$ and $H = Y \setminus f(X \setminus V)$ containing A and B such that $f^{-1}(G) \subset U$ and $f^{-1}(H) \subset V$. Clearly, G and H are disjoint ρ -open subsets of Y . Hence, Y is strongly ρ -normal. $\square$

Definition 2.25. (i) A space X is said to be strongly ρ -regular if for each ρ -closed set F and each point $x \notin F$, there exists disjoint ρ -open sets U and V in X such that $x \in U$ and $F \subset V$.

(ii) A space X is said to be almost ρ -regular if for each regular closed subset F and each point $x \notin F$, there exists disjoint ρ -open sets U and V in X such that $x \in U$ and $F \subset V$.

Theorem 2.26. If f is a completely ρ -irresolute, ρ^* -closed injection of an almost ρ -regular space X onto a space Y , then Y is strongly ρ -regular space.

Proof. Let F be a ρ -closed subset of Y and let $y \notin F$. Then, $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is almost ρ -regular space, there exists disjoint ρ -open sets U and V in X such that $f^{-1}(y) \in U$ and $f^{-1}(F) \subset V$. Since f is ρ^* -closed and by Theorem 2.22, there exist ρ -open sets $G = Y \setminus f(X \setminus U)$ such that $f^{-1}(G) \subset U$, $y \in G$ and $H = Y \setminus f(X \setminus V)$ such that $f^{-1}(H) \subset V$, $F \subset H$. Clearly, G and H are disjoint ρ -open subsets of Y . Hence, Y is strongly ρ -regular space.

3. Weakly ρ -irresolute functions

Definition 3.1. A function $f : X \rightarrow Y$ is said to be weakly ρ -irresolute if for each point $x \in X$ and each $V \in \rho O(Y, f(x))$, there exists a $U \in \rho O(X, x)$ such that $f(U) \subset \rho\text{-cl}(V)$.

Example 3.2. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is weakly ρ -irresolute.

It is evident that every ρ -irresolute function is weakly ρ -irresolute but the converse is not true.

Example 3.3. Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\phi, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is clearly weakly ρ -irresolute but not ρ -irresolute. Observe that for the ρ -closed set $V = \{c\}$ in (Y, σ) , $f^{-1}(V) = \{c\}$ is not ρ -closed in (X, τ) .

Definition 3.4. A space (X, τ) is called a ρ - T_s space [4] if every ρ -closed (resp. ρ -open) set is closed (resp. open).

Theorem 3.5. A function $f : X \rightarrow Y$ is weakly ρ -irresolute if the graph function, defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly ρ -irresolute.

Proof. Let $x \in X$ and $V \in \rho O(Y, f(x))$. Then $X \times V$ is a ρ -open set of $X \times Y$ containing $g(x)$. Since g is weakly ρ -irresolute, there exists $U \in \rho O(X, x)$ such that $g(U) \subset \rho\text{-cl}(X \times V) \subset X \times \rho\text{-cl}(V)$. Therefore, we have $f(U) \subset \rho\text{-cl}(V)$. $\square$

Lemma 3.6. A locally indiscrete ρ - T_s topological space is ρ - T_2 if and only if for each pair of distinct points $x, y \in X$, there exist $U \in \rho O(X, x)$ and $V \in \rho O(X, y)$ such that $\rho\text{-cl}(U) \cap \rho\text{-cl}(V) = \phi$.

Proof. This follows immediately from the definitions of locally indiscrete ρ - T_s space and Lemma 1.5. $\square$

Theorem 3.7. If a locally indiscrete ρ - T_s space Y is ρ - T_2 space and $f : X \rightarrow Y$ is a weakly ρ -irresolute injection, then X is ρ - T_2 .

Proof. Let x, y be any two distinct points of X . Since f is injective, we have $f(x) \neq f(y)$. Since Y is locally indiscrete ρ - T_s and ρ - T_2 space, by Lemma 3.6 there exists $V \in \rho O(Y, f(x))$ and $W \in \rho O(Y, f(y))$ such that $\rho\text{-cl}(V) \cap \rho\text{-cl}(W) = \phi$. Since f is weakly ρ -irresolute, there exists $G \in \rho O(X, x)$ and $H \in \rho O(X, y)$ such that $f(G) \subset \rho\text{-cl}(V)$ and $f(H) \subset \rho\text{-cl}(W)$. Hence we obtain $G \cap H = \phi$. This shows that X is ρ - T_2 . $\square$

Definition 3.8. A function $f : X \rightarrow Y$ is said to have strongly ρ -closed graph if for each $(x, y) \in (X \times Y) - G(f)$, there exist $U \in \rho O(X, x)$ and $V \in \rho O(Y, y)$ such that $[\rho\text{-cl}(U) \times \rho\text{-cl}(V)] \cap G(f) = \phi$.

Theorem 3.9. If a function $f : X \rightarrow Y$ is weakly ρ -irresolute, injective and $G(f)$ is strongly ρ -closed, then X is ρ - T_2 .

Proof. Let x, y be a pair of distinct points of X . Since f is injective, $f(x) \neq f(y)$

and $(x, f(y)) \notin G(f)$. Since $G(f)$ is strongly ρ -closed, there exist $G \in \rho O(X, x)$ and $V \in \rho O(Y, f(y))$ such that $f(\rho\text{-cl}(G)) \cap \rho\text{-cl}(V) = \phi$. Since f is weakly ρ -irresolute, there exists $H \in \rho O(X, y)$ such that $f(H) \subset \rho\text{-cl}(V)$. Hence we have $f(\rho\text{-cl}(G)) \cap f(H) = \phi$; hence $G \cap H = \phi$. This shows that X is $\rho\text{-}T_2$. $\square$

4. Conclusion

The authors introduce the classes of completely ρ -irresolute functions and weakly ρ -irresolute functions. Properties of completely ρ -irresolute functions and weakly ρ -irresolute functions are investigated. In the literature, many topologists investigated several irresoluteness for some questions in topology.

References

- [1] N. Biswas, *Characterizations of semi-continuous functions*, Atti. Accad. Naz. Lince. Rend. Cl Sci. Fis. Mat. Natur., **48**(1970), 399-402.
- [2] S. G. Crossley and S. K. Hildebrand, *Semi-topological properties*, Fund. Math., **74**(1972), 233-254.
- [3] C. Devamanoharan, S. Pious Missier and S. Jafari, *On ρ -closed sets in topological spaces*, (submitted).
- [4] C. Devamanoharan, S. Pious Missier and S. Jafari, *On ρ -continuous functions in topological spaces*, (submitted).
- [5] C. Devamanoharan, S. Pious Missier and S. Jafari, *On ρ -homeomorphisms in topological spaces*, (submitted).
- [6] K. Dłaska, N. Ergun and M. Ganster, *Countable S -closed spaces*, Math. Slovaca, **44**(1994), 337-348.
- [7] E. Ekici, *Completely p -irresolute functions*, Analele Univ. de Vest din Timisoara, Seria Mat.-Inf., XLII, (2)(2004), 17-25.
- [8] E. Ekici and T. Noiri, *On $\gamma\alpha$ -continuous functions*, Demonstratio Mathematica, **4**(2005), 943-951.
- [9] E. Ekici, *Generalization of perfectly continuous, Regular set-connected and clopen functions*, Acta. Math. Hungar., **107**(3)(2005), 193-206.
- [10] E. Ekici and S. Jafari, *On a weaker form of complete irresoluteness*, Bol. Soc. Paran. Mat., **26**(1-2)(2008), 81-87.
- [11] N. Ergun, *On nearly paracompact spaces*, Istanbul, Univ. Fen, Mec. Ser. A, **45**(1980), 65-87.
- [12] S. Jafari, T. Noiri, N. Rajesh and M. L. Thivagar, *Another generalization of closed sets*, Kochi J. Math., **3**(2008), 25-38.
- [13] N. Levine, *Strong continuity in topological spaces*, Amer. Math. Monthly, **67**(1960), 269.

- [14] N. Levine, *Semi-open sets, semi-continuity in topological spaces*, Amer. Math. Monthly, **70**(1963), 36-41.
- [15] P. E. Long and L. L. Herrington, *Basic properties of regular-closed functions*, Rend. Circ. Mat. Palermo, **27**(1978), 20-28.
- [16] G. D. Maio, *S-closed spaces, S-sets and S-continuous functions*, Acad. Sci. Torino, **118**(1984), 125-134.
- [17] A. S. Mashour, M. E. Abd El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings*, Proc. Math. phys. Soc. Egypt., **53**(1982), 47-53.
- [18] M. Mrsevic, *On pairwise R_0 and pairwise R_1 bitopological spaces*, Bull. Math. Soc. Sci. Math. R. S. Roumanie, **30**(78)(1986), 141-148.
- [19] T. Noiri and G. D. Maio, *Properties of α -compact spaces*, Rend. Circ. Mat. Palermo., Ser(II), (1998), 359-369.
- [20] M. K. Singal, A. R. Singal and A. Mathur, *On nearly compact spaces*, Boll. UMI, **4**(1969), 702-710.
- [21] M. H. Stone, *Application of the theory Boolean rings to general topology*, Trans. Amer. Math. Soc., **41**(1937), 375-381.
- [22] T. Thompson, *S-closed spaces*, Proc. Amer. Math. Soc., **60**(1976), 335-338.
- [23] M. K. R. S. Veerakumar, *$\hat{g}$ -closed sets in topological spaces*, Bull Allahabad. Soc., **18**(2003), 99-112.
- [24] M. K. R. S. Veerakumar, *$\#g$ -semi-closed sets in topological spaces*, Antarctica J. Maths, **2**(2)(2005), 201-202.
- [25] M. K. R. S. Veerakumar, *Between $g\sim$ -closed and g -closed sets*, Antarctica J. Maths, **3**(1)(2006), 43-65.