On Generalized ϕ -Recurrent LP-Sasakian Manifolds

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ABSTRACT. In this paper we studied generalized ϕ -recurrent and generalized concircular ϕ -recurrent LP-Sasakian manifolds.

1. Introduction

In 1977, T. Takahashi[4], introduced the notion of locally ϕ -symmetric Sasakian manifolds and obtained some interesting properties. Some authors like U.C. De and G. Pathak[8], Venkatesha and C.S. Bagewadi[9], A.A. Shaikh and U.C. De[10] have extended this notion to 3-dimentional Kenmotsu, trans-Sasakian, LP-Sasakian manifolds respectively.

In present paper we studied generalized ϕ -recurrent and generalized concircular ϕ -recurrent LP-Sasakian manifolds. The paper is organized as follows: In section 2, we give a brief account of LP-Sasakian manifolds. In section 3, we studied generalized ϕ -recurrent LP-Sasakian manifolds. At first it is shown that a generalized ϕ -recurrent LP-Sasakian manifold is an Einstein manifold. Then we have shown that in a generalized ϕ -recurrent LP-Sasakian manifold the characteristic vector field ξ and the vector field $\rho + \sigma$ associated with 1-form $\alpha + \beta$ are in opposite direction. In section 4, we studied generalized concircular ϕ -recurrent LP-Sasakian manifolds. We first obtain the relation between the 1-forms α and β , then we prove that it is an Einstein manifold. Finally we have shown that the characteristic vector field ξ and the vector fields ρ, σ associated to the 1-forms α, β respectively are in opposite direction.

2. Preliminaries

An n -dimensional differentiable manifold M_n is called an LP-Sasakian manifolds [1], [2] if it admits a (1,1) tensor field ϕ , a vector field ξ , a 1-form η and a Lorentzian metric g which satisfy

$$(1) \quad \eta(\xi) = -1, \phi^2(X) = X + \eta(X)\xi,$$

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Received April 14, 2009; revised September 10, 2009; accepted September 20, 2009.

2000 Mathematics Subject Classification: 53C05, 53C20, 53C25, 53D15.

Key words and phrases: Generalized ϕ -recurrent, generalized concircular ϕ -recurrent, LP-Sasakian manifold, Einstein manifold.

$$(2) \quad g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y),$$

$$(3) \quad g(X, \xi) = \eta(X), \nabla_X \xi = \phi X,$$

$$(4) \quad (\nabla_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g .

It can easily be seen that in an LP-Sasakian manifold, the following relations hold

$$(5) \quad \phi\xi = 0, \eta(\phi X) = 0,$$

$$(6) \quad \text{rank}(\phi) = n - 1.$$

Further in an LP-Sasakian manifold the following relations also hold[2],[3]

$$(7) \quad \begin{aligned} (a) \quad R(X, Y, \xi) &= \eta(Y)X - \eta(X)Y, \\ (b) \quad R(\xi, X, Y) &= g(X, Y)\xi - \eta(Y)X, \end{aligned}$$

$$(8) \quad S(X, \xi) = (n - 1)\eta(X),$$

$$(9) \quad R(X, \xi, \xi) = -X - \eta(X)\xi,$$

$$(10) \quad \eta(R(X, Y, Z)) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y),$$

$$(11) \quad S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y),$$

$$(12) \quad (\nabla_X \eta)(Y) = g(X, \phi Y) = g(\phi X, Y),$$

for any vector fields X, Y, Z . where $R(X, Y, Z)$ is the curvature tensor, S is the Ricci tensor.

3. Generalized ϕ -recurrent LP-Sasakian Manifolds.

Analogous of consideration of generalized recurrent manifolds[5], we give the following definition.

Definition 3.1. An LP-Sasakian manifold is called generalized ϕ -recurrent if its curvature tensor R satisfies the condition

$$(13) \quad \phi^2((\nabla_W R)(X, Y, Z)) = \alpha(W)R(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y],$$

where α and β are two 1-forms, β is non-zero and these are defined by

$$(14) \quad \alpha(W) = g(W, \rho), \beta(W) = g(W, \sigma),$$

and ρ, σ are vector fields associated with 1-forms α, β respectively.

If the 1-form β in (13) becomes zero, then the manifold reduces to a ϕ -recurrent LP-Sasakian manifold. ϕ -recurrent LP-Sasakian manifolds have been studied by Al-Aqeel, De and Ghosh[13].

From (13) using (1), we have

$$(15) \quad (\nabla_W R)(X, Y, Z) + \eta((\nabla_W R)(X, Y, Z))\xi \\ = \alpha(W)R(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y],$$

from which it follows that

$$(16) \quad g((\nabla_W R)(X, Y, Z), U) + \eta((\nabla_W R)(X, Y, Z))\eta(U) \\ = \alpha(W)g(R(X, Y, Z), U) + \beta(W)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)].$$

Let $\{e_i\}$, $i = 1, 2, \dots, n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (16) and taking summation over i , $1 \leq i \leq n$, we get

$$(17) \quad (\nabla_W S)(Y, Z) + \sum_{i=1}^n \eta((\nabla_W R)(e_i, Y, Z))\eta(e_i) \\ = \alpha(W)S(Y, Z) + (n-1)\beta(W)g(Y, Z).$$

The second term of L.H.S. in (17) by putting $Z = \xi$ assumes the form

$$g((\nabla_W R)(e_i, Y, \xi), \xi)g(e_i, \xi),$$

which is denoted by E . In this case E vanishes. Namely we have

$$g((\nabla_W R)(e_i, Y, \xi), \xi) = g(\nabla_W R(e_i, Y, \xi), \xi) - g(R(\nabla_W e_i, Y, \xi), \xi) \\ - g(R(e_i, \nabla_W Y, \xi), \xi) - g(R(e_i, Y, \nabla_W \xi), \xi),$$

at $p \in M$. Since $\{e_i\}$ is an orthonormal basis, so $\nabla_X e_i = 0$ at p . Using (7), we obtain

$$g(R(e_i, \nabla_W Y, \xi), \xi) = g(\nabla_W Y, \xi)g(e_i, \xi) - g(\xi, e_i)g(\nabla_W Y, \xi) = 0.$$

Thus we obtain

$$g((\nabla_W R)(e_i, Y, \xi), \xi) = g(\nabla_W R(e_i, Y, \xi), \xi) - g(R(e_i, Y, \nabla_W \xi), \xi).$$

In virtue of $g(R(e_i, Y, \xi), \xi) = g(R(\xi, \xi, Y), e_i) = 0$, we have

$$g(\nabla_W R(e_i, Y, \xi), \xi) + g(R(e_i, Y, \xi), \nabla_W \xi) = 0,$$

which implies

$$g((\nabla_W R)(e_i, Y, \xi), \xi) = -g(R(e_i, Y, \xi), \nabla_W \xi) - g(R(e_i, Y, \nabla_W \xi), \xi).$$

Hence we reach

$$\begin{aligned} E &= - \sum_{i=1}^n \{g(R(\phi W, \xi, Y), e_i)g(\xi, e_i) + g(R(\xi, \phi W, Y), e_i)g(\xi, e_i)\} \\ &= -\{g(R(\phi W, \xi, Y), \xi) + g(R(\xi, \phi W, Y), \xi)\} = 0. \end{aligned}$$

Replacing Z by ξ in (17) and using (8), we get

$$(18) \quad (\nabla_W S)(Y, \xi) = (n-1)\alpha(W)\eta(Y) + (n-1)\beta(W)\eta(Y).$$

Now, we have

$$(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi).$$

Using (8) and (3) in the above relation, it follows that

$$(19) \quad (\nabla_W S)(Y, \xi) = (n-1)g(W, \phi Y) - S(Y, \phi W).$$

In view of (18) and (19), we obtain

$$(20) \quad (n-1)g(W, \phi Y) - S(Y, \phi W) = (n-1)\alpha(W)\eta(Y) + (n-1)\beta(W)\eta(Y).$$

Replacing Y by ϕY in (20) and then using (1), (5) and (11), we get

$$(21) \quad S(Y, W) = (n-1)g(Y, W),$$

for all Y, W . This leads the following.

Theorem 3.1. *A generalized ϕ -recurrent LP-Sasakian manifold is an Einstein manifold.*

Two vector fields P and Q are said to be codirectional if $P = fQ$, where f is a non-zero scalar. That is $g(P, X) = fg(Q, X)$ for all X .

Now from (13), we have

$$(22) \quad (\nabla_W R)(X, Y, Z) = -\eta((\nabla_W R)(X, Y, Z))\xi + \alpha(W)R(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y].$$

Then by use of second Bianchi's identity and (22), we get

$$\begin{aligned} (23) \quad &\alpha(W)\eta(R(X, Y, Z)) + \alpha(X)\eta(R(Y, W, Z)) + \alpha(Y)\eta(R(W, X, Z)) \\ &+ \beta(W)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\ &+ \beta(X)[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\ &+ \beta(Y)[g(X, Z)\eta(W) - g(W, Z)\eta(X)] = 0. \end{aligned}$$

By virtue of (10), we obtain from (23)

$$(24) \quad \begin{aligned} & [\alpha(W) + \beta(W)][g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\ & + [\alpha(X) + \beta(X)][g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\ & + [\alpha(Y) + \beta(Y)][g(X, Z)\eta(W) - g(W, Z)\eta(X)] = 0. \end{aligned}$$

Putting $Y = Z = e_i$ in (24) and taking summation over i , $1 \leq i \leq n$, we get

$$(25) \quad \{\alpha(W) + \beta(W)\}\eta(X) = \{\alpha(X) + \beta(X)\}\eta(W),$$

for all vector fields X, W .

Replacing X by ξ in (25), it follows that

$$(26) \quad \alpha(W) + \beta(W) = -\eta(W)[\eta(\rho) + \eta(\sigma)],$$

for any vector field W , where $\alpha(\xi) = g(\xi, \rho) = \eta(\rho)$ and $\beta(\xi) = g(\xi, \sigma) = \eta(\sigma)$.

From (25) and (26) we can state that the following.

Theorem 3.2. *In a generalized ϕ -recurrent LP-Sasakian manifold the characteristic vector field ξ and the vector field $\rho + \sigma$ associated to the 1-form $\alpha + \beta$ are in opposite direction.*

In view of (3) and (7) it can be easily seen that in an LP-Sasakian manifold the following relation holds

$$(27) \quad (\nabla_W R)(X, Y, \xi) = g(W, \phi Y)X - g(W, \phi X)Y - R(X, Y, \phi W).$$

By virtue of (10), it follows from (27), that

$$(28) \quad \eta((\nabla_W R)(X, Y, \xi)) = 0.$$

Now assume that X, Y and Z are (local) vector fields such that $(\nabla X)_p = (\nabla Y)_p = (\nabla Z)_p = 0$ for a fixed point p of M_n . By Ricci identity for ϕ [6]

$$-(R(X, Y) \cdot \phi Z) = (\nabla_X \nabla_Y \phi)Z - (\nabla_Y \nabla_X \phi)Z.$$

We have at the point p ,

$$-R(X, Y, \phi Z) + \phi R(X, Y, Z) = \nabla_X((\nabla_Y \phi)Z) - \nabla_Y((\nabla_X \phi)Z).$$

Using (4), we have

$$\begin{aligned} & -R(X, Y, \phi Z) + \phi R(X, Y, Z) \\ & = \nabla_X \{g(Y, Z)\xi + \eta(Z)Y + 2\eta(Y)\eta(Z)\xi\} \\ & \quad - \nabla_Y \{g(X, Z)\xi + \eta(Z)X + 2\eta(X)\eta(Z)\xi\} \\ & = g(Y, Z) \nabla_X \xi + (\nabla_X \eta)(Z)Y + 2(\nabla_X \eta)(Y)\eta(Z)\xi \\ & \quad + 2(\nabla_X \eta)(Z)\eta(Y)\xi + 2\eta(Y)\eta(Z) \nabla_X \xi \\ & \quad - g(X, Z) \nabla_Y \xi - (\nabla_Y \eta)(Z)X - 2(\nabla_Y \eta)(X)\eta(Z)\xi \\ & \quad - 2(\nabla_Y \eta)(Z)\eta(X)\xi - 2\eta(X)\eta(Z) \nabla_Y \xi. \end{aligned}$$

In view of (3) and (12) the above equation becomes

$$(29) \quad \begin{aligned} R(X, Y, \phi Z) = & -g(\phi X, Z)Y - g(Y, Z)\phi X + g(\phi Y, Z)X + g(X, Z)\phi Y \\ & - 2g(X, \phi Z)\eta(Y)\xi - 2\eta(Y)\eta(Z)\phi X \\ & + 2g(Y, \phi Z)\eta(X)\xi + 2\eta(X)\eta(Z)\phi Y + \phi R(X, Y, Z), \end{aligned}$$

for any $X, Y, Z \in T_p M$. From (27) and (29), it follows that

$$(30) \quad \begin{aligned} (\nabla_W R)(X, Y, \xi) = & g(Y, W)\phi X - g(X, W)\phi Y + 2g(X, \phi W)\eta(Y)\xi \\ & + 2\eta(Y)\eta(W)\phi X - 2g(Y, \phi W)\eta(X)\xi \\ & - 2\eta(X)\eta(W)\phi Y - \phi R(X, Y, W). \end{aligned}$$

In view of (15) and (28), we obtain from (30)

$$\begin{aligned} \alpha(W)R(X, Y, \xi) + \beta(W)[g(Y, \xi)X - g(X, \xi)Y] \\ = g(Y, W)\phi X - g(X, W)\phi Y + 2g(X, \phi W)\eta(Y)\xi + 2\eta(Y)\eta(W)\phi X \\ - 2g(Y, \phi W)\eta(X)\xi - 2\eta(X)\eta(W)\phi Y - \phi R(X, Y, W). \end{aligned}$$

In view of (7) and (26), the above equation becomes

$$(31) \quad \begin{aligned} -\eta(W)[\eta(\rho) + \eta(\sigma)][\eta(Y)X - \eta(X)Y] \\ = g(Y, W)\phi X - g(X, W)\phi Y + 2g(X, \phi W)\eta(Y)\xi + 2\eta(Y)\eta(W)\phi X \\ - 2g(Y, \phi W)\eta(X)\xi - 2\eta(X)\eta(W)\phi Y - \phi R(X, Y, W). \end{aligned}$$

Hence if X and Y are orthogonal to ξ , then (31) reduces to

$$(32) \quad \phi R(X, Y, W) = g(Y, W)\phi X - g(X, W)\phi Y.$$

operating ϕ on both sides of (32) and using (1), we get

$$R(X, Y, W) = g(Y, W)X - g(X, W)Y.$$

This leads the following.

Theorem 3.3. *A generalized ϕ -recurrent LP-Sasakian manifold is a space of constant curvature provided that X and Y are orthogonal to ξ .*

4. Generalized concircular ϕ -recurrent LP-Sasakian manifolds.

Analogous of consideration of generalized recurrent manifolds[5], we give the following definition.

Definition 4.1. An LP- Sasakian manifold is called generalized concircular ϕ -recurrent if its concircular curvature tensor C ,

$$(33) \quad C(X, Y, Z) = R(X, Y, Z) - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]$$

satisfies the condition

$$(34) \quad \phi^2((\nabla_W C)(X, Y, Z)) = \alpha(W)C(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y],$$

where α and β are defined as (14) and r is the scalar curvature.

If the 1-form β in (34) becomes zero, then the manifold reduces to a concircular ϕ -recurrent manifolds. Concircular ϕ -recurrent LP-Sasakian manifolds has been studied by Venkatesha and Bagewadi[14].

Let us consider a generalized concircular ϕ -recurrent LP-Sasakian manifold. Then by virtue of (1) and (34), we have

$$(35) \quad \begin{aligned} (\nabla_W C)(X, Y, Z) + \eta((\nabla_W C)(X, Y, Z))\xi \\ = \alpha(W)C(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y], \end{aligned}$$

from which it follows that

$$(36) \quad \begin{aligned} g((\nabla_W C)(X, Y, Z), U) + \eta((\nabla_W C)(X, Y, Z))\eta(U) \\ = \alpha(W)g(C(X, Y, Z), U) + \beta(W)[g(Y, Z)g(X, U) - g(X, Z)g(Y, U)]. \end{aligned}$$

Let $\{e_i\}$, $i = 1, 2, \dots, n$ be orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (36) and taking summation over i , $1 \leq i \leq n$, we get

$$(37) \quad \begin{aligned} (\nabla_W S)(X, U) - \frac{W(r)}{n}g(X, U) + (\nabla_W S)(X, \xi)\eta(U) - \frac{W(r)}{n}\eta(X)\eta(U) \\ = \alpha(W)[S(X, U) - \frac{r}{n}g(X, U)] + (n-1)\beta(W)g(X, U). \end{aligned}$$

Replacing U by ξ in (37) then using (1) and (8), we have

$$(38) \quad \alpha(W)[(n-1) - \frac{r}{n}]\eta(X) + (n-1)\beta(W)\eta(X) = 0.$$

Putting $X = \xi$ in (38), we obtain

$$(39) \quad [(n-1) - \frac{r}{n}]\alpha(W) + (n-1)\beta(W) = 0.$$

Theorem 4.1. *Let (M_n, g) be a generalized concircular ϕ -recurrent LP-Sasakian manifold then*

$$[(n-1) - \frac{r}{n}]\alpha(W) + (n-1)\beta(W) = 0.$$

Now putting $X = U = e_i$ in (36) and taking summation over i , $1 \leq i \leq n$, we get

$$(40) \quad (\nabla_W S)(Y, Z) = - \sum_{i=1}^n g((\nabla_W R)(e_i, Y, Z), \xi) g(e_i, \xi) \\ + \frac{W(r)}{n} g(Y, Z) - \frac{W(r)}{n(n-1)} [g(Y, Z) + \eta(Y)\eta(Z)] \\ + \alpha(W)[S(Y, Z) - \frac{r}{n} g(Y, Z)] + (n-1)\beta(W)g(Y, Z).$$

Replacing Z by ξ in (40) and using (38), we have

$$(41) \quad (\nabla_W S)(Y, \xi) = \frac{W(r)}{n} \eta(Y).$$

Now, we have

$$(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi).$$

Using (3) and (8) in the above relation, it follows that

$$(42) \quad (\nabla_W S)(Y, \xi) = (n-1)g(\phi Y, W) - S(\phi Y, W).$$

In view of (41) and (42)

$$(43) \quad S(Y, \phi W) = (n-1)g(Y, \phi W) - \frac{W(r)}{n} \eta(Y).$$

Replacing Y by ϕY in (43) then using (2), (5) and (11), we obtain

$$(44) \quad S(Y, W) = (n-1)g(Y, W).$$

This leads to the following theorem.

Theorem 4.2. *A generalized concircular ϕ -recurrent LP-Sasakian manifold is an Einstein manifold.*

Now from (35), we have

$$(45) \quad (\nabla_W C)(X, Y, Z) = \alpha(W)C(X, Y, Z) + \beta(W)[g(Y, Z)X - g(X, Z)Y] \\ - \eta((\nabla_W C)(X, Y, Z))\xi.$$

This implies

$$(46) \quad (\nabla_W R)(X, Y, Z) = -\eta((\nabla_W R)(X, Y, Z))\xi + \alpha(W)R(X, Y, Z) \\ + \frac{W(r)}{n(n-1)} [g(Y, Z)X - g(X, Z)Y + g(Y, Z)\eta(X)\xi \\ - g(X, Z)\eta(Y)\xi] - [\frac{r}{n(n-1)}\alpha(W) - \beta(W)] \\ [g(Y, Z)X - g(X, Z)Y].$$

Now from (46) and Bianchi's second identity, we have

$$\begin{aligned}
 (47) \quad & \alpha(W)\eta(R(X, Y, Z)) + \alpha(X)\eta(R(Y, W, Z)) + \alpha(Y)\eta(R(W, X, Z)) \\
 &= -\left\{\beta(W) - \frac{r}{n(n-1)}\alpha(W)\right\}[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\
 &\quad - \left\{\beta(X) - \frac{r}{n(n-1)}\alpha(X)\right\}[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\
 &\quad - \left\{\beta(Y) - \frac{r}{n(n-1)}\alpha(Y)\right\}[g(X, Z)\eta(W) - g(W, Z)\eta(X)].
 \end{aligned}$$

By virtue of (10), we obtain from (47), that

$$\begin{aligned}
 (48) \quad & \alpha(W)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\
 &+ \alpha(X)[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\
 &+ \alpha(Y)[g(X, Z)\eta(W) - g(W, Z)\eta(X)] \\
 &= -\left\{\beta(W) - \frac{r}{n(n-1)}\alpha(W)\right\}[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] \\
 &\quad - \left\{\beta(X) - \frac{r}{n(n-1)}\alpha(X)\right\}[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] \\
 &\quad - \left\{\beta(Y) - \frac{r}{n(n-1)}\alpha(Y)\right\}[g(X, Z)\eta(W) - g(W, Z)\eta(X)].
 \end{aligned}$$

Putting $Y = Z = e_i$ in (48) and taking summation over i , $1 \leq i \leq n$, we get

$$\begin{aligned}
 (49) \quad & (a) \quad \alpha(W)\eta(X) = \alpha(X)\eta(W), \\
 & (b) \quad \beta(W)\eta(X) = \beta(X)\eta(W),
 \end{aligned}$$

for all vector fields X, W . Replacing X by ξ in (49), we get

$$\begin{aligned}
 (50) \quad & (a) \quad \alpha(W) = -\eta(W)\eta(\rho), \\
 & (b) \quad \beta(W) = -\eta(W)\eta(\sigma),
 \end{aligned}$$

for all vector field W .

From (49) and (50), we can state the following.

Theorem 4.3. *In a generalized concircular ϕ -recurrent LP-Saskian manifold the characteristic vector field ξ and the vector fields ρ, σ associated to the 1-forms α, β respectively are in opposite direction and the 1-forms α, β are given by (50).*

Acknowledgment. The authors are thankful to the referee for his valuable suggestions in the improvement of the paper.

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