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A Discrete Model of Brucellosis Happened in Korean Livestock Farms

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Abstract. In this paper we introduce a discrete model of brucellosis happened in Korean livestock farms and numerically analyze its dynamical features. To do it, we consider parameters data supported by Livestock Cooperatives. To control brucellosis, we investigate the relationship among key parameters, as applications of our model. We hope that our model may be used to reduce brucellosis in Korean livestock farms.

1. Introduction

The purpose of this paper is to introduce a mathematical model for brucellosis happened in Korean livestock farms. As a zoonosis, Brucellosis caused by bacteria of the genus Brucella is an infectious disease that is able to be transmitted from other animals, both wild and domestic, to humans or from humans to animals. Now, it is primarily a worldwide distributed disease of domestic animals (goats, pigs, cattle, dogs, etc) and humans, specially in developing countries([15]). Based on data([16]) of National Veterinary Research and Quarantine Service(NVRQS) in Korea, $8,944$ heads of $920,219$ cows in 183,284 livestock farms turned out to be brucellosis-positive by a serum test until September, 2007. The outbreak of brucellosis in livestock farms increases steadily from month to month. NVRQS thus reforms a system of precautionary measures and tries to reduce brucellosis in farms. In order to control the spread of brucellosis among cattle and people effectively, we have to set up an appropriate mathematical model for brucellosis happened in Korean livestock farms and to find some effective ways to control the spread of a disease based on the data of NVRQS. This is our research aim as an application of Mathematics.

This paper is organized as follows. In Section 2, we shortly describe Brucella disease in the biological point of view. In Section 3, a mathematical model for brucellosis is introduced in economic and biological point of view. In the last Section, we give numerical results of our model and concluding remarks.

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2. Biological summary for brucellosis

In most countries brucellosis is a notifiable disease. Overall brucellosis has an important worldwide impact on animal industries and human health. In humans, brucellosis can cause a range of symptoms that are similar to the flu and may include fever, sweats, headaches, back pains, and physical weakness. Severe infections of the central nervous systems or lining of the heart may occur. Brucellosis can also cause long-lasting symptoms that include recurrent fevers, joint pain, and fatigue. Because of these reasons, epidemiological researches on brucellosis have been proceeding for a long time([6], [7], [8], [9], [10], [11]).

Cattle affected with Brucella *abortus* have high incidences of abortions, or calve weak offspring, arthritic joints, and retention of after-birth, known as retained placenta. In cattle, brucellosis is primarily a disease of the female, the cow. In the cow, the organism localizes in the udder, uterus, and lymph nodes adjacent to the uterus. The infected cows exhibit symptoms that may include abortion during the last third of pregnancy, retained afterbirth, and weak calves at birth. Infected cows usually abort only once. Some infected cows will not exhibit ant clinical symptoms of the disease and give birth to normal calves. There are two main causes for spontaneous abortion in animals. The first is due to erythrotol, which can promote infections in the fetus and placenta. Second is due to the lack of anti-Brucella activity in the amniotic fluid. Males can also harbor the bacteria in their reproductive tracts, namely seminal vesicles, ampullae, testicles, and epididymides. Dairy herds in the USA are tested at least once a year with the Brucella Milk Ring Test(BRT)([4]). Cows that are confirmed to be infected are often killed. In the United States, veterinarians are required to vaccinate all young stock, thereby further reducing the chance of zoonotic transmission. Canada declared their cattle herd brucellosis-free on September 19, 1985. Brucellosis ring testing of milk and cream, as well as testing of slaughter cattle, ended April 1, 1999. Monitoring continues through auction market testing, standard disease reporting mechanisms, and testing of cattle being qualified for export to countries other than the USA([5]).

Brucellosis cause some problems for people as fevers, sweating, weakness, anemia, headaches, depression and muscular and bodily pain. Humans are generally infected in one of three ways : eating or drinking something that is contaminated with Brucella, breathing in the organism, or having the bacteria enter the body through skin wounds. The most common way to be infected is by eating or drinking contaminated milk products. If the milk is not pasteurized, these bacteria can be transmitted to persons who drink the milk or eat cheeses made it. Inhalation of Brucella organisms is not a common route of infection, but it can be a significant hazard for people in certain occupations, such as those working in laboratories where the organism is cultured([14]).

3. Mathematical model

In this section, we illustrate a mathematical model of brucellosis happened in

Korean livestock farms. To describe our model, we have to consider an economical situation in a farm, because farmers grow cows to get an economical profit. It means that they continuously buy calves and then after growing it, they sell it. This kind of dynamical behavior may be described by using *inflow* and *outflow* concepts. We may thus assume that there is a capacity for growing cow population in a farm. That is, a capacity K in the farm is a maximal number of cow and calves lived together in a farm.

Related to brucellosis in a farm, it is seriously happened when female cow delivery calves. To set up a model for brucellosis, we divide the adult cow population into two sub-populations: the susceptible-cow population, S, and the latently infected cow population, L, having brucellosis virus but not exposing virus. We also divide the calve population into two sub-populations depending on infection: the susceptible-calves population, J, and the latently infected calves population, JL.

Typically to describe an epidemic model, continuous flow systems are often used. But, after we examine a data given by Livestock Cooperatives, Our finding from the given data is that there is a periodic time in female cow's lifecycle. For instance, to delivery calves, female cows spend 10 months and then for 4 months, female cows spend in resting before getting a pregnancy. After a female cow deliveries calves by 3 times, they should be slaughtered. Due to the periodicity of dynamical behaviors, a discrete model may be possible. Now, we may denote $S_{n\tau}$ by the susceptible-cow population at the time $n\tau$, where τ indicates a time period with 14. To more simplify a notation, let $S_{n\tau} \equiv S_n$.

To determine the susceptible-cow population, S_{n+1}, at the time $n+1$, we have to consider the infected and slaughtered population from S_n at time n and the surviving susceptible-calves population from selling, because calves grow adults after spending one time period. The equation for S_{n+1} is described by the following :

$$S_{n+1} = (1-\alpha)S_n\left(1-\frac{r(L_n+JL_n)}{K}\right) + (1-\delta)J_n\left(1-\frac{r(L_n+JL_n)}{K}\right), \tag{3.1}$$

where α is the parameters indicating the rate of slaughtering Korean cow per a period, δ for the rate of selling calves per a period, and r for the parameter of infection rate. Infection of susceptible cows and calves is depending on contact of contaminations including cow's blood and encounters of latently infected cow and calves. It implies that infection rate is proportional to latently infected populations per a capacity.

The latently infected cow population, L_{n+1}, at the $n+1$ will be determined by summing up the remaining latently infected calves population after selling calves, $(1-\alpha)JL_n$, surviving latently infected cow population from slaughter and destroying, $(1-\alpha-\beta)L_n$, and newly infected populations. We thus get the following equation :

$$(3.2)\qquad L_{n+1} = (1-\alpha-\beta)L_n + (1-\delta)JL_n + \frac{(1-\alpha)S_n r(L_n+JL_n)}{K} + \frac{r(1-\delta)J_n(L_n+JL_n)}{K}.$$

The susceptible-calves population, J_{n+1}, at the time n is counted from inflows from inside and outside of a farm. That is, newly delivering calves, $(1-\alpha)pS_n$, attributed to the pregnancy of remained susceptible female cows at the time n or newly delivering calves from latently infected cows which is related to the rate h of the latently infected cow delivering healthy calves and the abortion rate γ, and new susceptible calves σE moved from outside of a farm, where σ is the probability that newly inserted calves in a farm did not have brucellosis. We get the following equation :

$$(3.3)\qquad J_{n+1} = (1-\alpha)pS_n + (1-\alpha)h(1-\gamma)pL_n + \sigma E.$$

The population of latently infected calves may be described by inflows of latently infected cows and of latently infected calves from outside of a farm.

$$(3.4)\qquad JL_{n+1} = (1-\alpha)(1-h)(1-\gamma)pL_n + (1-\sigma)E$$

As a way of inserting new calves into a farm, we may consider the following system :

$$(3.5)\qquad E = \begin{cases} K-T, & \text{if } T \le K, \\ 0, & \text{if } T > K, \end{cases}$$

where T is a total sum of populations, *i.e.*, $T = S(i)+L(i)+J(i)+JL(i)$. Combining the equations from Eq.(3.1) to Eq.(3.5), we finally get the mathematical model of brucellosis happened in Korean livestock farms as following :

$$\begin{aligned} S_{n+1} &= (1-\alpha)S_n\left(1-\frac{r(L_n+JL_n)}{K}\right) + (1-\delta)J_n\left(1-\frac{r(L_n+JL_n)}{K}\right), \\ L_{n+1} &= (1-\alpha-\beta)L_n + (1-\delta)JL_n \\ &\quad + \frac{(1-\alpha)S_n r(L_n+JL_n)}{K} + \frac{r(1-\delta)J_n(L_n+JL_n)}{K}, \\ J_{n+1} &= (1-\alpha)pS_n + (1-\alpha)h(1-\gamma)pL_n + \sigma(K-T), \\ JL_{n+1} &= (1-\alpha)(1-h)(1-\gamma)pL_n + (1-\sigma)(K-T). \end{aligned}$$

4. Numerical results and concluding remarks

To apply our model into real situation, we may use the following data supported by Korean Livestock Cooperatives(see the table 1).

Parameter	Value
α (the rate of slaughtering)	1/3
β (the rate of destroying Latently infected)	5/105
γ (the rate of abortion)	0.05
δ (the rate of selling calves)	85/105
p (the pregnant rate)	0.8

Table 1: Parameter data for a discrete model for brucellosis

Using this data set, the parameters r, h, σ are unknown. The parameter h is a biological factor. It would be got from a biological experiment. But, the other two parameters r and σ are controlled by farmers and governmental policy.

First, we numerically calculate orbits for several parameter settings. For instance, Fig. 1 shows behaviors of populations. Numerically we find that all orbits converges to an endemic equilibrium point for each parameter setting.

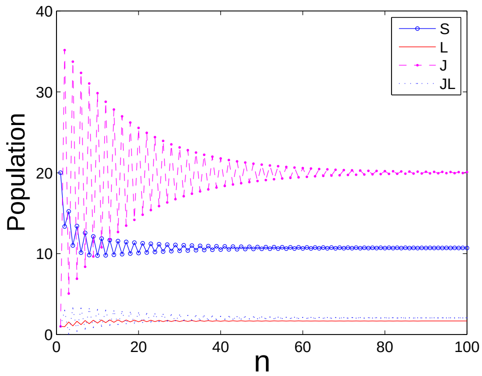

Figure 1: An orbit for $r = 0.3$, $h = 0.4$, and $\sigma = 0.9$

To numerically show that all orbits converges to an endemic equilibrium point for each parameter setting, we fix $h = 0.2$ and then we calculate the maximal eigenvalues for each endemic equilibrium point corresponding each parameters r and σ. To do it, we calculate the Jacobian matrix $\mathbf{J}$ evaluated a fixed point :

$$\mathbf{J} = \begin{pmatrix} (1-\alpha)P^* & Q^* & (1-\delta)P^* & Q^* \\ (1-\alpha)S^* & (1-\alpha-\beta)+R^* & (1-\delta)S^* & (1-\delta)+R^* \\ \{(1-\alpha)p-\sigma\} & \{(1-\alpha)h(1-\gamma)p-\sigma\} & -\sigma & -\sigma \\ \sigma-1 & \{(1-\alpha)(1-h)(1-\gamma)p+\sigma-1\} & \sigma-1 & \sigma-1 \end{pmatrix},$$

where

$$P^* = 1 - \frac{r(L_n + JL_n)}{K}, \qquad Q^* = -\frac{r}{K}\{(1-\alpha)S_n + (1-\delta)J_n\},$$
$$R^* = \frac{r}{K}\{(1-\alpha)S_n + (1-\delta)J_n\}, \quad S^* = \frac{r}{K}(L_n + JL_n).$$

Figure 2 shows the maximal eigenvalues. It means all eigenvalues are less than 1. We thus get that endemic equilibrium points are stable.

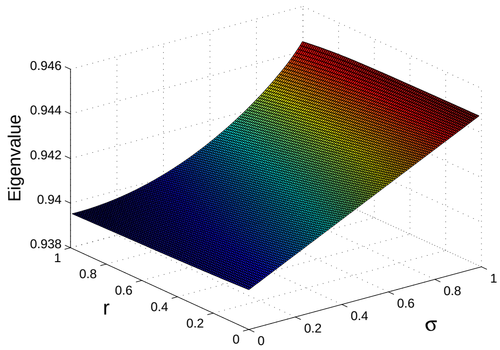

Figure 2: Maximal eigenvalues verse parameters r, σ and $h = 0.2$

The both of controlling parameters r and σ are important factors, but in certain circumstances, we have to decide a priority of these parameters in the sense of economical and political reasons. To check up this question, we compute the infected ratio $(L^* + JL^*)/K$, where L^* and JL^* are the latently infected cow and calve of endemic equilibrium point, respectively. In Fig. 3, the infected ratio is shown and this ratio is increasing as $r \to 1$ and $\sigma \to 0$.

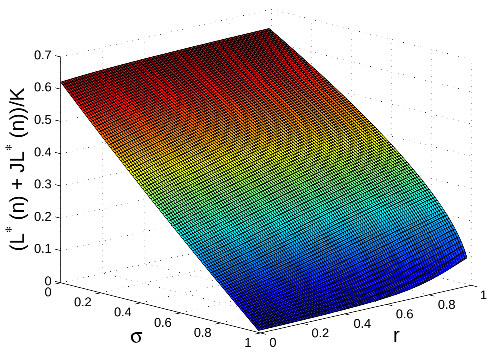

Figure 3: The ratio of infected populations verse parameters r, σ and $h = 0.2$

To detail investigate relationship between two parameters, we numerically calculate the infection ratio related to the infection parameter under a fixed inserting probability σ, as shown in Fig. 4. The infection ratio is approximately increasing about 0.15 related to r, but as shown in Fig. 5, the infection ratio related to σ is more quickly increasing. It means that in controlling brucellosis disease, we simultaneously focus on the two controlling parameters. But, we have to more pay attention to insert new susceptible calves into a farm.

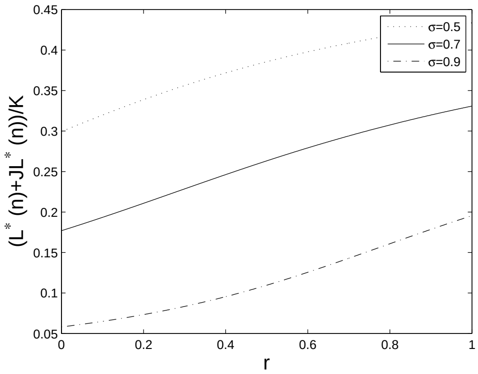

Figure 4: The ratio of infected populations verse the infection parameter r

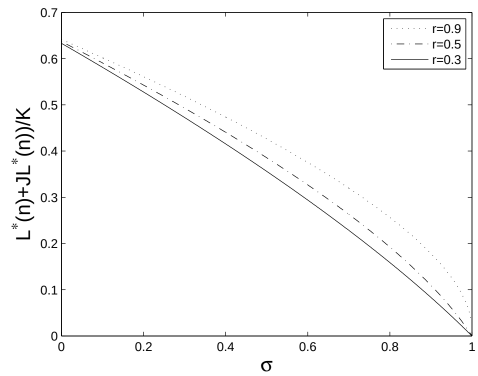

Figure 5: The ratio of infected populations verse the controlling parameter σ

In this paper, we introduce a new mathematical model for brucellosis happened in Korean livestock farms and discuss a controlling problem focusing on controlling parameters. In shorter and longer time, our suggested model may be useful to control brucellosis in Korean farm.

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