Exposed Bilinear Forms of $\mathcal{L}({}^2d_*(1, w)^2)$

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ABSTRACT. First we present the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}({}^2d_*(1, w)^2)^*$. Using this formula and results of [16] and [17], we show that every extreme bilinear form of the unit ball of $\mathcal{L}({}^2d_*(1, w)^2)$ is exposed.

1. Introduction

We write B_E for the closed unit ball of a real Banach space E and the dual space of E is denoted by E^* . $x \in B_E$ is called an *extreme point* of B_E if $y, z \in B_E$ with $x = \frac{1}{2}(y + z)$ implies $x = y = z$. $x \in B_E$ is called an *exposed point* of B_E if there is a $f \in E^*$ so that $f(x) = 1 = \|f\|$ and $f(y) < 1$ for every $y \in B_E \setminus \{x\}$. It is easy to see that every exposed point of B_E is an extreme point. We denote by $\exp B_E$ and $\text{ext} B_E$ the sets of exposed and extreme of B_E , respectively. For $n \geq 2$, we denote by $\mathcal{L}({}^nE)$ the Banach space of all continuous n -linear forms on E endowed with the norm $\|T\| = \sup_{\|x_k\|=1, 1 \leq k \leq n} |T(x_1, \dots, x_n)|$. $\mathcal{L}_s({}^nE)$ denotes the subspace of all continuous symmetric n -linear forms on E . A mapping $P : E \rightarrow \mathbb{R}$ is a continuous n -homogeneous polynomial if there exists $T \in \mathcal{L}_s({}^nE)$ such that $P(x) = T(x, \dots, x)$ for every $x \in E$. We denote by $\mathcal{P}({}^nE)$ the Banach space of all continuous n -homogeneous polynomials from E into $\mathbb{R}$ endowed with the norm $\|P\| = \sup_{\|x\|=1} |P(x)|$. For more details about the theory of multilinear mappings and polynomials on a Banach space, we refer to [7]. We will denote by $T((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + \frac{c}{2}(x_1y_2 + x_2y_1)$ and $P(x, y) = ax^2 + by^2 + cxy$ a symmetric bilinear form and a 2-homogeneous polynomial on a real Banach space of dimension 2 respectively. For $1 \leq p \leq \infty$, we let $l_p^2 = \mathbb{R}^2$ with the l_p -norm. Note that in ([6], Theorem 1, remark after Theorem 1, and Theorem 2) the following

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results are proved: (i) $\exp B_{\mathcal{P}(2l_1^2)} = \text{ext} B_{\mathcal{P}(2l_1^2)} \setminus \{\pm(x^2 - y^2 \pm 2xy)\}$;

(ii) $\exp B_{\mathcal{P}(2l_\infty^2)} = \text{ext} B_{\mathcal{P}(2l_\infty^2)} \setminus \{\pm(\frac{1}{2}x^2 - \frac{1}{2}y^2 \pm xy)\}$.

The author [11] characterized $\exp B_{\mathcal{P}(2l_p^2)}$ as follows: (i) If $1 < p < 2$, then $\exp B_{\mathcal{P}(2l_p^2)} = \text{ext} B_{\mathcal{P}(2l_p^2)}$;

(ii) If $2 < p < \infty$, then $\exp B_{\mathcal{P}(2l_p^2)} = \text{ext} B_{\mathcal{P}(2l_p^2)} \setminus \{\pm x^2, \pm y^2\}$.

We refer to ([1–6, 8–22] and references therein) for some recent work about extremal properties of multilinear mappings and homogeneous polynomials on some classical Banach spaces.

We denote the 2-dimensional real predual of the Lorentz sequence space with a positive weight $0 < w < 1$ by

$$d_*(1, w)^2 := \{(x, y) \in \mathbb{R}^2 : \|(x, y)\|_{d_*} := \max\{|x|, |y|, \frac{|x| + |y|}{1 + w}\}\}.$$

Very recently, the author [16] characterizes the extreme points of the unit ball of $\mathcal{L}(^2d_*(1, w)^2)$. The author [17] also proves that every extreme symmetric bilinear form of the unit ball of $\mathcal{L}_s(^2d_*(1, w)^2)$ is exposed.

In this paper we first present the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}(^2d_*(1, w)^2)^*$. Using the explicit formula for the norm of a (continuous) linear functional of $\mathcal{L}(^2d_*(1, w)^2)^*$ and results of [16] and [17], we prove that every extreme bilinear form of the unit ball of $\mathcal{L}(^2d_*(1, w)^2)$ is exposed.

2. The Results

If $T \in \mathcal{L}(^2d_*(1, w)^2)$, then $T((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1$ for some reals a, b, c, d .

Theorem 2.1. ([16], Theorem 2.1) *Let $T((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1, w)^2)$. Then there exists (unique) $T'((x_1, y_1), (x_2, y_2)) = a^*x_1x_2 + b^*y_1y_2 + c^*x_1y_2 + d^*x_2y_1 \in \mathcal{L}(^2d_*(1, w)^2)$ such that $a^*, b^*, c^*, d^* \in \{\pm a, \pm b, \pm c, \pm d\}$ with $a^* \geq b^* \geq 0, c^* \geq |d^*|$ and $\|T\| = \|T'\|$ and that T is extreme if and only if T' is extreme.*

Theorem 2.2. ([16], Theorem 2.2) *Let $T((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1, w)^2)$ with $a \geq |b|, c \geq |d|$. Then*

$$\|T\| = \max\{a + bw^2 + (c + d)w, a - bw^2 + (c - d)w, (a + b)w + c + dw^2, (a - b)w + c - dw^2\}.$$

Theorem 2.3. ([16], Theorem 2.4) *Let $S((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \mathcal{L}(^2d_*(1, w)^2)$ with $a \geq b \geq 0, c \geq |d|$. Then*

(a) Let $w < \sqrt{2} - 1$. S is extreme if and only if

$$\begin{aligned} S \in \{ & x_1x_2, x_1y_2, \frac{1}{1+w}(x_1x_2 + x_1y_2), \frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1), \\ & \frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1), \frac{1}{1+w^2}(wx_1x_2 + wy_1y_2 + x_1y_2 - x_2y_1), \\ & \frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1), \\ & \frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1-w-w^2)x_1y_2 - wx_2y_1), \\ & \frac{1}{(1+w)^2(1-w)}((1-w-w^2)x_1x_2 + wy_1y_2 + x_1y_2 - x_2y_1)\}. \end{aligned}$$

(b) Let $w = \sqrt{2} - 1$. Then S is extreme if and only if

$$\begin{aligned} S \in \{ & x_1x_2, x_1y_2, \frac{1}{\sqrt{2}}(x_1x_2 + x_1y_2), \frac{1}{2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1), \\ & \frac{\sqrt{2}}{4}((\sqrt{2}+1)(x_1x_2 + y_1y_2) + x_1y_2 - x_2y_1), \\ & \frac{\sqrt{2}}{4}(x_1x_2 + y_1y_2 + (\sqrt{2}+1)(x_1y_2 - x_2y_1))\}. \end{aligned}$$

(c) Let $w > \sqrt{2} - 1$. Then S is extreme if and only if

$$\begin{aligned} S \in \{ & x_1x_2, x_1y_2, \frac{1}{1+w}(x_1x_2 + x_1y_2), \frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1), \\ & \frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1), \\ & \frac{1}{1+w^2}(\frac{1-w}{1+w}(x_1x_2 + y_1y_2) + x_1y_2 - x_2y_1), \\ & \frac{1}{1+w^2}(x_1x_2 + y_1y_2 + \frac{1-w}{1+w}(x_1y_2 - x_2y_1)), \\ & \frac{1}{2+2w}(x_1x_2 + y_1y_2 + (2+w)x_1y_2 - \frac{1}{w}x_2y_1), \\ & \frac{1}{2+2w}((2+w)x_1x_2 + \frac{1}{w}y_1y_2 + x_1y_2 - x_2y_1)\}. \end{aligned}$$

Theorem 2.4. ([16], Theorem 2.5) $T \in \text{ext}B_{\mathcal{L}^2 d_*(1, w)^2}$ if and only if there exist $n \in \mathbb{N}$ and $S((x_1, y_1), (x_2, y_2)) = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \text{ext}B_{\mathcal{L}^2 d_*(1, w)^2}$ with $a \geq |b|, c \geq |d|$ such that $T((x_1, y_1), (x_2, y_2)) := S((u_1^{(n)}, v_1^{(n)}), (u_2^{(n)}, v_2^{(n)})) \circ \cdots \circ ((u_1^{(1)}, v_1^{(1)}), (u_2^{(1)}, v_2^{(1)}))$,

where

$$\begin{aligned} \text{for } j = 1, \dots, n, ((u_1^{(j)}, v_1^{(j)}), (u_2^{(j)}, v_2^{(j)})) \in \{((\pm x_1, \pm y_1), (\pm x_2, \pm y_2)), \\ ((\pm x_2, \pm y_2), (\pm x_1, \pm y_1)), ((\pm x_1, \pm y_1), (\pm y_2, \pm x_2)), ((\pm y_2, \pm x_2), \\ (\pm x_1, \pm y_1)), ((\pm y_1, \pm x_1), (\pm x_2, \pm y_2)), ((\pm x_2, \pm y_2), (\pm y_1, \pm x_1)), \\ ((\pm y_2, \pm x_2), (\pm y_1, \pm x_1)), ((\pm y_1, \pm x_1), (\pm y_2, \pm x_2))\}. \end{aligned}$$

Theorem 2.5. Let $f \in \mathcal{L}(^2d_*(1, w)^2)^*$ and $\alpha = f(x_1x_2), \beta = f(y_1y_2),$
 $\delta = f(x_1y_2), \gamma = f(x_2y_1).$

(a) Let $w < \sqrt{2} - 1$. Then

$$\begin{aligned} \|f\| = & \max\{|\alpha|, |\beta|, |\delta|, |\gamma|, \frac{1}{1+w}|\alpha + \gamma|, \frac{1}{1+w}|\alpha + \delta|, \\ & \frac{1}{1+w}|\beta + \gamma|, \frac{1}{1+w}|\beta + \delta|, \frac{1}{(1+w)^2}(|\alpha + \beta| + |\delta + \gamma|), \\ & \frac{1}{(1+w)^2}(|\alpha - \beta| + |\delta - \gamma|), \frac{1}{1+2w-w^2}(|\alpha + \beta| + |\delta - \gamma|), \\ & \frac{1}{1+2w-w^2}(|\alpha - \beta| + |\delta + \gamma|), \frac{1}{1+w^2}(|\alpha + \beta| + w|\delta - \gamma|), \\ & \frac{1}{1+w^2}(|\alpha - \beta| + w|\delta + \gamma|), \frac{1}{1+w^2}(|\delta + \gamma| + w|\alpha - \beta|), \\ & \frac{1}{1+w^2}(|\delta - \gamma| + w|\alpha + \beta|), \\ & \frac{1}{(1+w)^2(1-w)}(|\alpha + \beta| + |(1-w-w^2)\delta - w\gamma|) \\ & \frac{1}{(1+w)^2(1-w)}(|\alpha - \beta| + |(1-w-w^2)\delta + w\gamma|), \\ & \frac{1}{(1+w)^2(1-w)}(|\alpha + \beta| + |(1-w-w^2)\gamma - w\delta|) \\ & \frac{1}{(1+w)^2(1-w)}(|\alpha - \beta| + |(1-w-w^2)\gamma + w\delta|), \\ & \frac{1}{(1+w)^2(1-w)}(|\delta + \gamma| + |(1-w-w^2)\alpha - w\beta|), \\ & \frac{1}{(1+w)^2(1-w)}(|\delta - \gamma| + |(1-w-w^2)\alpha + w\beta|), \\ & \frac{1}{(1+w)^2(1-w)}(|\delta + \gamma| + |(1-w-w^2)\beta - w\alpha|), \\ & \frac{1}{(1+w)^2(1-w)}(|\delta - \gamma| + |(1-w-w^2)\beta + w\alpha|)\}. \end{aligned}$$

(b) Let $w = \sqrt{2} - 1$. Then

$$\begin{aligned} \|f\| = & \max\{|\alpha|, |\beta|, |\delta|, |\gamma|, \frac{1}{\sqrt{2}}|\alpha + \gamma|, \frac{1}{\sqrt{2}}|\alpha + \delta|, \frac{1}{\sqrt{2}}|\beta + \gamma|, \\ & \frac{1}{\sqrt{2}}|\beta + \delta|, \frac{\sqrt{2}}{4}((\sqrt{2} + 1)|\alpha + \beta| + |\delta - \gamma|), \\ & \frac{\sqrt{2}}{4}((\sqrt{2} + 1)|\delta + \gamma| + |\alpha - \beta|), \frac{\sqrt{2}}{4}((\sqrt{2} + 1)|\alpha - \beta| + |\delta + \gamma|), \\ & \frac{\sqrt{2}}{4}(|\alpha + \beta| + (\sqrt{2} + 1)|\delta - \gamma|)\}. \end{aligned}$$

(c) Let $\sqrt{2} - 1 < w$. Then

$$\begin{aligned} \|f\| = & \max\{|\alpha|, |\beta|, |\delta|, |\gamma|, \frac{1}{1+w}|\alpha + \gamma|, \frac{1}{1+w}|\alpha + \delta|, \\ & \frac{1}{1+w}|\beta + \gamma|, \frac{1}{1+w}|\beta + \delta|, \frac{1}{(1+w)^2}(|\alpha + \beta| + |\delta + \gamma|), \\ & \frac{1}{(1+w)^2}(|\alpha - \beta| + |\delta - \gamma|), \frac{1}{1+2w-w^2}(|\alpha + \beta| + |\delta - \gamma|), \\ & \frac{1}{1+2w-w^2}(|\alpha - \beta| + |\delta + \gamma|), \frac{1}{1+w^2}(\frac{1-w}{1+w}|\alpha + \beta| + |\delta - \gamma|), \\ & \frac{1}{1+w^2}(|\alpha - \beta| + \frac{1-w}{1+w}|\delta + \gamma|), \frac{1}{2+2w}(|\alpha + \beta| + |(2+w)\delta - \frac{1}{w}\gamma|), \\ & \frac{1}{2+2w}(|\alpha + \beta| + |(2+w)\gamma - \frac{1}{w}\delta|), \frac{1}{2+2w}(|(2+w)\alpha - \frac{1}{w}\beta| + |\delta + \gamma|), \\ & \frac{1}{2+2w}(|(2+w)\beta - \frac{1}{w}\alpha| + |\delta + \gamma|)\}. \end{aligned}$$

Proof. It follows from Theorems 2.3–4 since

$$\|f\| = \sup\{|f(T)| : T \in \text{ext}B_{\mathcal{L}^2 d_*(1, w)^2}\}.$$

□

Theorem 2.6. ([17], Theorem 2.3) *Let E be a real Banach space such that $\text{ext}B_E$ is finite. Suppose that $x \in \text{ext}B_E$ satisfies that there exists an $f \in E^*$ such that $f(x) = 1 = \|f\|$ and $|f(y)| < 1$ for every $y \in \text{ext}B_E \setminus \{x\}$. Then $x \in \text{exp}B_E$.*

Using Theorems 2.1–6, we classify the exposed bilinear forms of the unit ball of $\mathcal{L}^2 d_*(1, w)^2$.

Theorem 2.7. $\text{exp}B_{\mathcal{L}^2 d_*(1, w)^2} = \text{ext}B_{\mathcal{L}^2 d_*(1, w)^2}$.

Proof. Let $L = ax_1x_2 + by_1y_2 + cx_1y_2 + dx_2y_1 \in \text{ext}B_{\mathcal{L}^2 d_*(1, w)^2}$. By Theorems 2.1 and 2.5, we may assume that $a \geq |b|, c \geq |d|$. Now we can use Theorem 2.3.

Case 1: $w < \sqrt{2} - 1$

Claim: x_1x_2 is exposed

Let $f \in \mathcal{L}({}^2d_*(1, w)^2)^*$ and $\alpha = f(x_1x_2), \beta = f(y_1y_2), \delta = f(x_1y_2), \gamma = f(x_2y_1)$. Let $\alpha = 1, \beta = 0 = \delta = \gamma$. By Theorem 2.5(a), $f(x_1x_2) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq x_1x_2$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+w}(x_1x_2 + x_1y_2)$ is exposed

Let $\alpha = \frac{1+w}{2} = \delta, \beta = 0 = \gamma$. By Theorem 2.5(a), $f(\frac{1}{1+w}(x_1x_2 + x_1y_2)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{1+w}(x_1x_2 + x_1y_2)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)$ is exposed

Let $\alpha = \frac{1+w^2}{2}, \beta = \frac{1+w^2}{2} - \epsilon, \delta = w + \frac{\epsilon}{2} = \gamma$ for a sufficiently small $\epsilon > 0$. By Theorem 2.5(a), $f(\frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{(1+w)^2}(x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)$ is exposed

Let $\alpha = \frac{1}{2} = \beta, \delta = \frac{w}{2}, \gamma = -\frac{w}{2}$. By Theorem 2.5(a), $f(\frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{1+w^2}(x_1x_2 + y_1y_2 + wx_1y_2 - wx_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)$ is exposed

Let $w < \delta < \frac{1-w^2}{2}$ and $\alpha = \frac{1+2w-w^2-2\delta}{2}, \beta = \alpha, \gamma = -\delta$. By Theorem 2.5(a), $f(\frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{1+2w-w^2}(x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1)$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1-w-w^2)x_1y_2 - wx_2y_1)$ is exposed

Let $\alpha = \frac{1+\epsilon(-1+w+w^2)}{2} = \beta, \delta = w + \epsilon, \gamma = 0$. for a sufficiently small $\epsilon > 0$. By Theorem 2.5(a), $f(\frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1-w-w^2)x_1y_2 - wx_2y_1)) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{(1+w)^2(1-w)}(x_1x_2 + y_1y_2 + (1-w-w^2)x_1y_2 - wx_2y_1)$. By Theorem 2.6, it is exposed.

Case 2: $w = \sqrt{2} - 1$

By the similar argument as Case 1, $\pm x_1x_2, \pm \frac{1}{\sqrt{2}}(x_1x_2 + x_1y_2), \pm \frac{1}{2}[x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1]$ are exposed. It is enough to show that $\frac{\sqrt{2}}{4}[x_1x_2 + y_1y_2 + (\sqrt{2} + 1)(x_1y_2 - x_2y_1)]$ is exposed. Let $\alpha = 0 = \beta, \delta = 2 - \sqrt{2} = -\gamma$. By Theorem 2.5(b), $f(\frac{\sqrt{2}}{4}[x_1x_2 - y_1y_2 + (\sqrt{2} + 1)(x_1y_2 + x_2y_1)]) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{\sqrt{2}}{4}[x_1x_2 - y_1y_2 + (\sqrt{2} + 1)(x_1y_2 + x_2y_1)]$. By Theorem 2.6, it is exposed.

Case 3: $\sqrt{2} - 1 < w$

By the similar argument as Case 1, $\pm x_1x_2, \pm \frac{1}{1+w}(x_1x_2 + x_1y_2), \pm \frac{1}{(1+w)^2}[x_1x_2 + y_1y_2 + x_1y_2 + x_2y_1], \pm \frac{1}{1+2w-w^2}[x_1x_2 + y_1y_2 + x_1y_2 - x_2y_1]$ are exposed.

Claim: $\frac{1}{1+w^2}[x_1x_2 + y_1y_2 + \frac{1-w}{1+w}(x_1y_2 - x_2y_1)]$ is exposed

Let $\alpha = \frac{1+w^2}{2} = -\beta, \delta = \gamma$. By Theorem 2.5(c), $f(\frac{1}{1+w^2}[x_1x_2 + y_1y_2 +$

$\frac{1-w}{1+w}(x_1y_2 - x_2y_1)] = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{1+w^2}[x_1x_2 + y_1y_2 + \frac{1-w}{1+w}(x_1y_2 - x_2y_1)]$. By Theorem 2.6, it is exposed.

Claim: $\frac{1}{2+2w}[(2+w)x_1x_2 + \frac{1}{w}y_1y_2 + (x_1y_2 - x_2y_1)]$ is exposed

Let $\alpha = 1 - \epsilon, \beta = w^2, \delta = \frac{\epsilon(2+w)}{2} = -\gamma$ for a sufficiently small $\epsilon > 0$. By Theorem 2.5(c), $f(\frac{1}{2+2w}[(2+w)x_1x_2 + \frac{1}{w}y_1y_2 + (x_1y_2 - x_2y_1)]) = 1 = \|f\|$ and $|f(T)| < 1$ for every $T \in \text{ext}B_{\mathcal{L}({}^2d_*(1, w)^2)}$ with $T \neq \frac{1}{2+2w}[(2+w)x_1x_2 - \frac{1}{w}y_1y_2 + (x_1y_2 + x_2y_1)]$. By Theorem 2.6, it is exposed. $\square$

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